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WITH AN ORGANIZATION BY THE USE OF
MARKOV CHAINS

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Estimating the Human Capital Associated With an Organization
by the Use of Markov Chains*

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Recently, Lev and Schwartz, and Morse have discussed the concept of human capital and presented arguments indicating why the concept of human capital is of interest to accountants.¹ The purpose of this paper is to demonstrate how Markov Chains can be used to estimate the human capital values associated with an organization.

HUMAN CAPITAL

The concept of human capital has received considerable attention in the economic literature in recent years as attempts have been made to explain the dramatic increase that has taken place in the return on tangible assets and to measure the returns from investment in education. Schultz has noted that less capital tends to be employed relative to income as economic growth proceeds when only nonhuman capital is considered. But, he adds that we cannot infer that the stock of all capital has been decreasing relative to income. If we assume that a roughly constant ratio between all capital and income is being maintained, the decline in the ratio between nonhuman capital and income is simply a signal that human capital has been increasing relative to conventional capital and income.² Students of economics, after studying the concept of human capital, have come to the conclusion that "laborers have become capitalists not from a diffusion of the ownership of corporate stock, as folklore would have it, but from the acquisition of knowledge and skill that have economic value."³

¹All footnotes at end of paper.

According to Lev and Schwartz, the value of human capital embodied in a person at time τ is the present value of his remaining future earnings from employment.⁴ Extending this notion, the value of the human capital employed in an organization is the present value of the future earnings of its current employees.⁵ Under conditions of certainty, the value of the human capital employed in an organization as it exists at time τ is:⁶

$$C = \sum_{i=1}^N \int_{\tau}^T \frac{E_i(t)}{(1+r)^{t-\tau}} dt \quad (1)$$

where:

C = value of human capital employed in an organization;

N = number of individuals currently employed by the organization;

τ = current time;

T = highest time at which an individual currently employed leaves the organization;

$E_i(t)$ = all direct and indirect compensation given individual i at time t by the organization; and

r = time value of money.

With uncertainty, the value of the human capital employed in an organization at time τ is:

$$C^* = \sum_{i=1}^N \left[\int_{\tau}^T \frac{E_i^*(t)}{(1+r)^{t-\tau}} dt - \int_{\tau}^T P_i(j) \int_j^T \frac{E_i^*(t)}{(1+r)^{t-\tau}} dt dj \right] \quad (2)$$

where:

* refers to expected values;

$P_i(j)$ = conditional probability of individual i 's leaving the organization at time j given that he is employed by the organization at time τ .

The first portion of equation 2 within the brackets represents the expected present value of the gross earnings of individual i assuming he remains with the organization until retirement. The second portion of equation 2 within the brackets represents the present value of the gross earnings of individual i which he will not receive because of the probability of his leaving the organization before retirement.

HUMAN RESOURCE ACCOUNTING

The concept of human capital discussed in this paper is related to, but not the same as the concept of human assets. However, accounting for human capital and accounting for human assets both fall under the general caption of human resource accounting. According to Morse:

Human resource accounting has two components: human asset accounting and human capital accounting. Human asset accounting is concerned with the determination of the value of the human resources employed in an organization to the organization. Human capital accounting is concerned with determining the value of the human resources employed in an organization to the employees of that organization.

Thus, the general classification, human resource accounting, has two subclassifications: human asset accounting and human capital accounting. Each is concerned with a separate aspect of the total human resources employed in an organization. The total value of the human resources employed in an organization is equal to the value of the organization's human assets and its employees' human capital.⁷

Most of the research in the area of human resource accounting has dealt with the determination of the value of employees to an organization.⁸ However, Morse has pointed out that the value of the human assets employed in an organization is directly affected by the value of the human capital of the organization's employees.⁹ Additionally, Lev and Schwartz have suggested a number of inferences that decision makers could draw from reported values of human capital. These include inferences about the

labor intensity of an organization and about changes in the structure of the labor force.¹⁰ Later in this paper it will be shown that interferences about changes in turnover and the potential effects of change in pay scales can also be drawn from reported values of human capital.

It should be noted that Hekiman and Jones, and Flamholtz question the propriety of attempting to measure an individual's value by determining the present value of his future earnings.¹¹ Indeed, the gross value of the services of all individuals employed in an organization should exceed the value of their earnings. It is this excess that leads to the notion of human assets. Assuming an organization employs additional personnel until the value of their marginal product equals their marginal cost, only the last person hired will have a gross value precisely equal to the value of his earnings. The concept of human capital is more concerned with measuring the present value of employment costs than measuring the net value of all employees to an organization. Because of this concentration on costs, rather than on net values, it is akin to traditional accounting cost based measures. The concept of human capital differs from traditional accounting measures to the extent that the costs it deals with extend beyond periods covered by current contractual obligations. However, if the organization is a going concern, these future obligations to current employees must, ultimately, be met.

MARKOV APPROACH

While equation 2 is a conceptually satisfying description of the amount of human capital associated with an organization in an uncertain world, it is difficult to operationalize. Fortunately, the literature contains numerous examples of the application of a mathematical technique

to similar situations involving uncertainty and future cash flows. In a path-breaking article, Cyert, Davidson, and Thompson applied Markov Chains to the problem of estimating the allowance for doubtful accounts.¹² More recently, Shank applied Markov Chains to the problem of income determination at a tree farm.¹³ Similar models have also been successfully employed for manpower planning.¹⁴ Accordingly, it seems appropriate to use such a model to estimate the human capital associated with an organization.

In the remaining sections of this paper, the problem of estimating the human capital associated with an organization in an uncertain world will be modeled in terms of an absorbing Markov Chain. Following that, an example of how the model can be applied will be presented and the limitations of the model will be discussed.

The Model

The objective of the model is to estimate the value of the human capital currently employed within an organization given the current pay scale, turnover, and movement of current employees through various service states and wage classifications within the organization. Intermediate steps in obtaining this objective include 1) estimating the average number of years a current employee will spend in various service states, age categories, and wage classifications given his current service state, age category, and wage classification; and 2) estimating the average number of man years all current employees will spend in various service states, age categories, and wage classifications given their current service states, age categories, and wage classifications.

The movement of employees through various service states, age categories, and wage classifications can be formulated as a transition probability

matrix, as in Figure 1. The r_{ij} represent the probability of an employee leaving the organization during the next time period due to death, retirement, or other cause given his current service state, age category and wage classification. The q_{ij} represent the probability of an employee going from one service state, age category, and wage classification to another during the next time period. The information required to complete the matrix can be obtained from records of employee movement during the past year.¹⁵

The transition probability matrix presented in Figure 1 can be partitioned into two sub-matrices, R and Q. Then standard matrix operations can be performed on matrix Q to determine the average number of years a current employee will spend in various service states, age categories, and wage classifications given his current status:¹⁶

$$N = (I - Q)^{-1} \quad (3)$$

where:

N = the fundamental matrix for an absorbing Markov Chain. The entries in N give the mean number of years an employee will be in each nonabsorbing state for each possible nonabsorbing starting state;

Q = the matrix of transition probabilities between nonabsorbing states; and

I = an identity matrix of the same order as matrix Q.

The average number of man years all current employees will spend in various service states, age categories, and wage classifications can now be found by postmultiplying a row vector that indicates the number of employees in each service state, age category, and wage classification by N.

FIGURE I

Transition Probability Matrix of Employee Movement Through Service States, Age Categories, and Wage Classifications

Service State I.....Service State II.....	
Termination	Age A Age B
Death	Wage 1
	Wage 2
Retirement	Wage 3
	Other
r_{11} r_{12}	$q_{11}q_{12}$
r_{31}	q_{1m}
r_{ij} ...	q_{ij}
r_{n1}	q_{nb}
	q_{nm}

$$M = TN \quad (4)$$

where:

M = a row vector indicating the average number of man years all current employees will spend in each service state, age category, and wage classification; and

T = a row vector indicating the number of employees in each service state, age category, and wage classification.

Finally, the total compensation that will be paid to all employees can be determined by postmultiplying M by a column vector of compensation paid to individual employees in each service state, age category, and wage classification.

$$C_{nd}^* = TP \quad (5)$$

where:

C_{nd}^* = the estimated value of the human capital employed in an organization when the time value of money is zero; and

P = a column vector of compensation paid to individual employees in each service state, age category, and wage classification.

An immediate problem with the above procedure for determining the total value of the human capital associated with an organization is the assumption of a zero time value of money. Fortunately, this problem can be overcome by premultiplying Q by a scalar representing the present value of a dollar received one period from now.¹⁷

$$C^* = T(I-dQ)^{-1}P \quad (6)$$

where:

d = a scalar representing the present value of a dollar received one period from now.

Assuming an appropriate discount rate can be determined, equation 6 can be used to estimate the value of the human capital currently employed within an organization given the current pay scale, turnover, and movement of employees through various service states and wage classifications.

Example

The basic transition probability matrix used in the example is presented in Figure 2. This matrix is simplified to conserve space. There is only one wage classification for each service state and age category, and the number of service states and age categories are limited. Vectors for the number of employees and yearly wage rates in each service state and age category are presented in Figure 3. The yearly wage rate includes all indirect compensation such as employer social security taxes and pension payments. For clarity of analysis, it is assumed that all pension benefits are funded and vested yearly.

Solving equation 3 for the information contained in Figure 2, the average number of years a current employee will spend in various service states, age categories, and wage classifications given his current status is obtained. This information is presented in Figure 4. Solving equation 4, the average number of man years all current employees will spend in various service states, age categories, and wage classifications is obtained. This information is presented in Figure 5. Using equation 5, the total compensation that will be paid to all employees is determined to be \$116,060,000. If consideration is given to the time value of money, as in equation 6, the total human capital associated with the organization is \$62,735,000 when the discount rate is 10 percent, and \$46,185,000 when the discount rate is 20 percent.

FIGURE 2*
Transition Probability Matrix of Employee Movement: An Example

	Age 25-34				Age 35-44			Age 45-54			Age 55-64		
	Termination	Unskilled	Semi-skilled	Skilled	Professional	Unskilled	Semi-skilled	Skilled	Professional	Unskilled	Semi-skilled	Skilled	Professional
Age 25-34													
Unskilled	.40	.43	.13		.01	.02	.01						
Semi-skilled	.35		.40	.20	.01		.03	.01					
Skilled	.10			.82	.01			.07					
Professional	.05				.87				.08				
Age 35-44													
Unskilled	.35					.35	.25			.03	.02		
Semi-skilled	.20						.65	.09		.04	.02		
Skilled	.05							.86			.09		
Professional	.07								.86		.07		
Age 45-54													
Unskilled	.10									.75	.07	.07	.01
Semi-skilled	.09									.80	.01	.10	
Skilled	.05										.86	.09	
Professional	.05										.86		.09
Age 55-64													
Unskilled	.15									.84	.01		
Semi-skilled	.14										.85	.01	
Skilled	.14											.86	
Professional	.12												.88

*The numbers in this table are contrived and a close analysis of them may reveal that they are unrealistic.

FIGURE 3

Vectors of the Number of Employees and Yearly Wage Rates

$T^T =$	600	6,000
	70	6,500
	10	8,000
	30	12,000
	400	6,300
	80	6,800
	25	10,000
	35	15,000
	200	6,400
	50	7,000
	30	12,000
	40	17,000
	100	6,500
	55	7,000
	30	13,000
	50	22,000

FIGURE 5

Average Number of Man-Years All Current Employees Will Spend
in Each Nonabsorbing State

	Man-Years
Age 25-34	
Unskilled	1,052.60
Semi-skilled	344.74
Skilled	438.60
Professional	372.00
Age 35-44	
Unskilled	647.77
Semi-skilled	750.89
Skilled	905.21
Professional	462.57
Age 45-54	
Unskilled	877.73
Semi-skilled	772.16
Skilled	958.63
Professional	517.00
Age 55-64	
Unskilled	1,009.00
Semi-skilled	1,007.20
Skilled	902.49
Professional	804.42

(In the final draft of this paper, the above example will include an analysis of the effects of changes in turnover, wage scales, and the number of employees on human capital.)

Limitations

Implementation of the above model for estimating the value of the human capital currently employed in an organization is hindered by a number of assumptions. Some of these assumptions can be overcome by modifying the model to fit a particular situation. Others are fundamental in nature and restrict the model's applicability. The assumptions underlying the model, represented by equation 6, are:

- 1) Employees remain in their current age category, service state, and wage classification throughout the first year;
- 2) All transitions take place at the end of each year;
- 3) Employees are paid at the beginning of each year;
- 4) An appropriate discount rate can be determined;
- 5) Except for cost of living adjustments, pay scales are stable;
- 6) All pension benefits are funded and vested at the beginning of each period;
- 7) Sufficient data is available to develop the transition probability matrix; and
- 8) While the transition probability matrix may change as a result of future changes in the organization or the external environment, it is basically stable from period to period.

Problems caused by the first three assumptions can be overcome by mathematical manipulation and, hence, should cause little difficulty in attempts to implement the model. Selecting the appropriate discount rate poses a more serious problem. While a discussion of the issues surrounding

the selection of an appropriate discount rate is beyond the scope of this paper, it should be noted that the appropriate discount rate does not have to allow for inflation if employee wages are tied to inflation. This is so because the model only considers current wage rates.

The restriction of the model to current wage rates causes the value of the human capital associated with an organization to change whenever there is a change in wage rates. This is not a deficiency in the model. Indeed, the disclosure of the amount of such a change may provide useful information to investors, employees, and other interested parties. For example, an increase in wage rates would cause an increase in the value of the human capital associated with an organization. This increase indicates that a larger amount of the future revenues of the organization will now go to the employees of the organization than was previously anticipated. If future wage increases can be anticipated, as a result of current contractual arrangements, the model can be manipulated to take them into account.

The assumption relating to the funding and vesting of pension benefits severely restricts the model. These restrictions can be relaxed only by greatly increasing the mathematical complexity of the model or excluding the organization's contributions to pensions from the model and treating them as a separate issue.

Assumptions seven and eight are the most fundamental in any type of Markov analysis of personnel movement, and they restrict the model's usefulness to "large," "stable" organizations. A large organization is necessary in order to obtain current information on the transition probabilities between various age categories, service states, and wage classifications. A stable organization is necessary in order to predict future

employee movements. Two comments should be made about the stability of the organization. First, the stability assumption only refers to stability in the absence of changes in the organization (such as management style) or the external environment (such as a depression). If such changes take place and they affect employee turnover or other aspects of the transition probability matrix, the model is designed to provide information about the effects of such changes on human capital. Second, the stability assumption does not require that, in the absence of changes in the organization or the external environment, no changes take place in the transition probability matrix at any time in the future. As the discount rate increases the model becomes insensitive to future changes.

(In the final draft of this paper, an examination will be made of the sensitivity of the model to changes in the transition probability matrix under varying discount rates.)

In any event, stability is the most important assumption underlying the model. Vroom and Mac Crimmon have suggested that a χ^2 test could be used to determine whether or not the underlying process was stable over time.¹⁸ While the stability assumption may severely limit the number of businesses that can use the model presented in this paper, previous research has shown that the stability required by the model is present in a number of governmental organizations.¹⁹ This suggests that units of government might report the value of the human capital they employ in various activities to the public.

CONCLUSIONS

Markov models, which have been used for manpower planning and estimating future cash flows, may also be useful in estimating the human capital associated with "large," "stable" organizations. A major advantage of the Markov approach is that it allows for movement between service states and wage classifications and provides information about the impact of changes in wages, turnover, and personnel policies on the value of the human capital associated with an organization. The major limitations of the Markov approach are the initial data requirements and the need for stability in the absence of changes in the organization or the external environment.

FOOTNOTES

¹B. Lev and A. Schwartz, "On the Use of the Economic Concept of Human Capital in Financial Statements," The Accounting Review (January, 1971), pp. 103-112. W. Morse, "A Note on the Relationship Between Human Assets and Human Capital," The Accounting Review (July, 1973), pp. 589-593.

²T. W. Schultz, "Investment in Human Capital," The American Economic Review (March, 1961), p. 5.

³H. G. Johnson, "The Political Economy of Opulence," Canadian Journal of Economics and Political Science (November, 1960), p. _____.

⁴Lev and Schwartz, p. 105.

⁵The restriction to current employees is similar to the approach taken by Philips in his definition of "pension liability" as

the value (at that point in time when we are measuring the liability) of the employer's obligation to make future pension payments to current and retired employees. This includes the obligation to make future payments based on or "related to" services yet to be performed (unearned benefits) but excludes anticipated pension payments to those to be employed in the future.

Like Philips' definition of "pension liability," the above definition of the "human capital employed in an organization" cannot be demonstrated to be a "true" definition. G. Edward Philips, "Pension Liabilities and Assets," The Accounting Review (January, 1968), p. 10.

⁶Morse, p. 590.

⁷Morse, p. 593.

⁸For a comprehensive bibliography of previous research in the area of human resource accounting, see the "Report of the Committee on Human Resource Accounting," The Accounting Review (Supplement to Vol. XLVII, 1973), pp. 182-185.

⁹Morse, pp. 590-593.

¹⁰Lev and Schwartz, pp. 107-109.

¹¹J. S. Hekimian and C. H. Jones, "Put People On Your Balance Sheet," Harvard Business Review (January-February, 1967), pp. 107-113. E. Flamholtz, "On the Use of the Economic Concept of Human Capital in Financial Statements: A Comment," The Accounting Review (January, 1972), pp. 148-152.

¹²R. M. Cyert, H. J. Davidson, and G. L. Thompson, "Estimation of the Allowance for Doubtful Accounts by Markov Chains," Management Science (April, 1962), pp. 287-303.

¹³J. K. Shank, "Income Determination Under Uncertainty: An Application of Markov Chains," The Accounting Review (January, 1971), pp. 57-74.

¹⁴J. W. Walker, "Models in Manpower Planning," in Manpower Planning and Programming, E. H. Barack and J. W. Walker, editors, (Allyn and Bacon, 1972), pp. 130-142. V. H. Vroom and K. R. Mac Crimmon, "Toward A Stochastic Model of Managerial Careers," Administrative Science Quarterly (June, 1968), pp. 26-46. K. M. Rowland and M. G. Sovereign, "Markov-Chain Analysis of Internal Manpower Supply," Industrial Relations (October, 1969), pp. 88-99. W. R. Dill, D. P. Gaver, and W. L. Weber, "Models and Modeling for Manpower Planning," Management Science, (December, 1966), pp. B-142-167.

¹⁵Such information should be developed by the personnel department for the purpose of manpower planning. Accordingly, the additional costs involved in using it for accounting purposes should be minimal. Problems caused by data requirements are discussed further in the section titled "Limitations."

¹⁶See: J. Kemeny; A. Schleifer, Jr.; J. Snell; and G. Thompson, Finite Mathematics (Prentice Hall, 1972), pp. 222-229. W. J. Morse, "Similarities Underlying Accounting Applications of Matrix Algebra," unpublished paper (The University of Illinois, 1974).

¹⁷C* is the present value of an infinite series of wage payments made at the beginning of each year:

$$C^* = TP + dTQP + d^2TQ^2P + d^3TQ^3P + \dots + d^kTQ^kP + \dots$$

$$C^* = T(I + dQ + d^2Q^2 + d^3Q^3 + \dots + d^kQ^k + \dots)P$$

$$C^* = T(I - dQ)^{-1}P$$

If it is assumed that the wage payments are made at the end of the year:

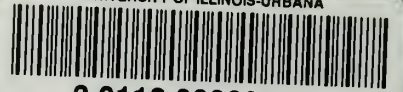
$$C^* = dT(I - dQ)^{-1}P$$

See: J. Shank, Matrix Methods in Accounting (Addison-Wesley, 1972), pp. 89-95, and Cyert, Davidson, and Thompson, pp. _____.

¹⁸Vroom and Mac Crimmon, pp. 42-43.

¹⁹Walker, pp. 136-137.

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